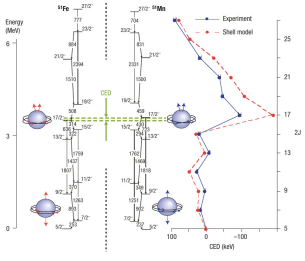
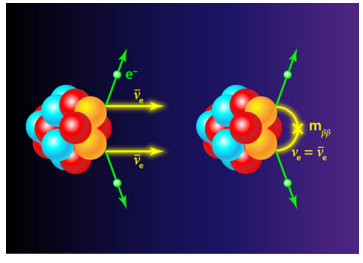
Nuclear structure within the Shell Model: new frontiers

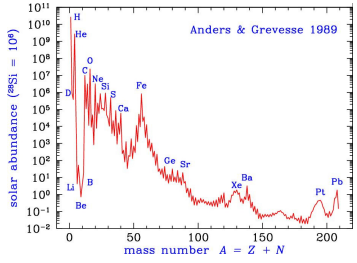
Dao Duy Duc and Frédéric Nowacki
(IPHC-Strasbourg)

GANIL - Autrans Méaudre en Vercors

September 29th, 2021

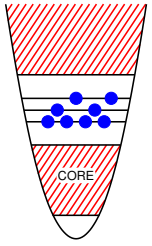






cnrs IN2P3 Les deux infinis
IPHC Institut Pluridisciplinaire Hubert Curien Strasbourg
Université de Strasbourg

Shell model description of the nucleus



$$\mathcal{H}_{\text{eff}}\Psi_{\text{eff}} = E\Psi_{\text{eff}}$$

$$\mathcal{H}_{\text{eff}} = \mathcal{H}_m + \mathcal{H}_M$$

(N. Smirnova's talk this morning)

Exact Diagonalization

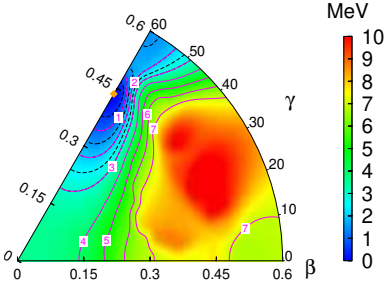
- Valence spaces
- Effective interactions
- Gigantic dimension: $D \sim \binom{d_\pi}{p}.\binom{d_\nu}{n}$

(Strasbourg codes ANTOINE, NATHAN)

Generator Coordinate Method (GCM)

- Deformed Hartree-Fock:

$$|\Phi\rangle = \prod_{i=1}^{A} a_i^\dagger |0\rangle$$

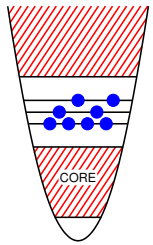


- Rotational symmetry restoration: $\mathcal{P}^J_{MK}|\Phi(q)\rangle \propto |J,M\rangle$
- Mixing of non-orthogonal states by the GCM:

$$|\Psi^{(J,M)}\rangle = \sum f^{(J)}(q,K)\mathcal{P}^J_{MK}|\Phi(q)\rangle$$

$$\sum_{q,K}\left(H^{(J)}_{K'K}(q',q) - E^{(J)}\, N^{(J)}_{K'K}(q',q)\right) x\ f^{(J)}(q,K) = 0$$

Shell model description of the nucleus



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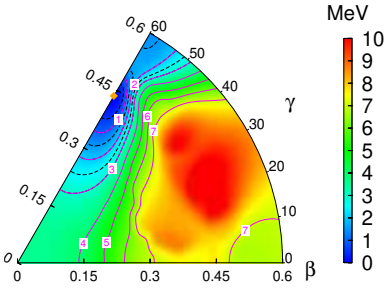
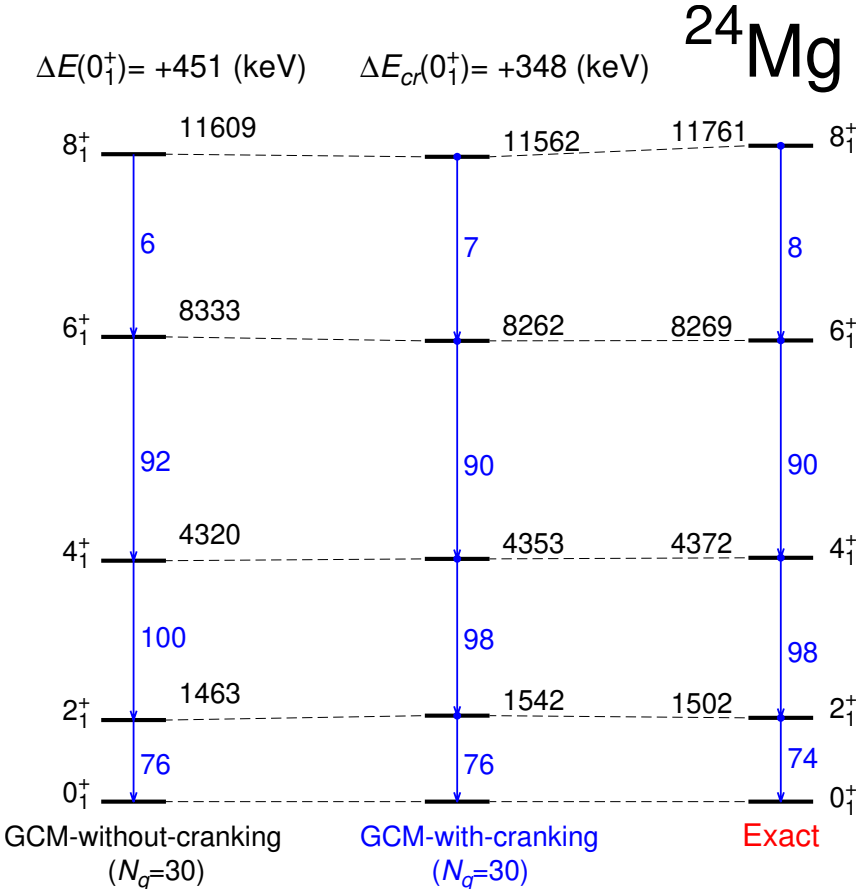


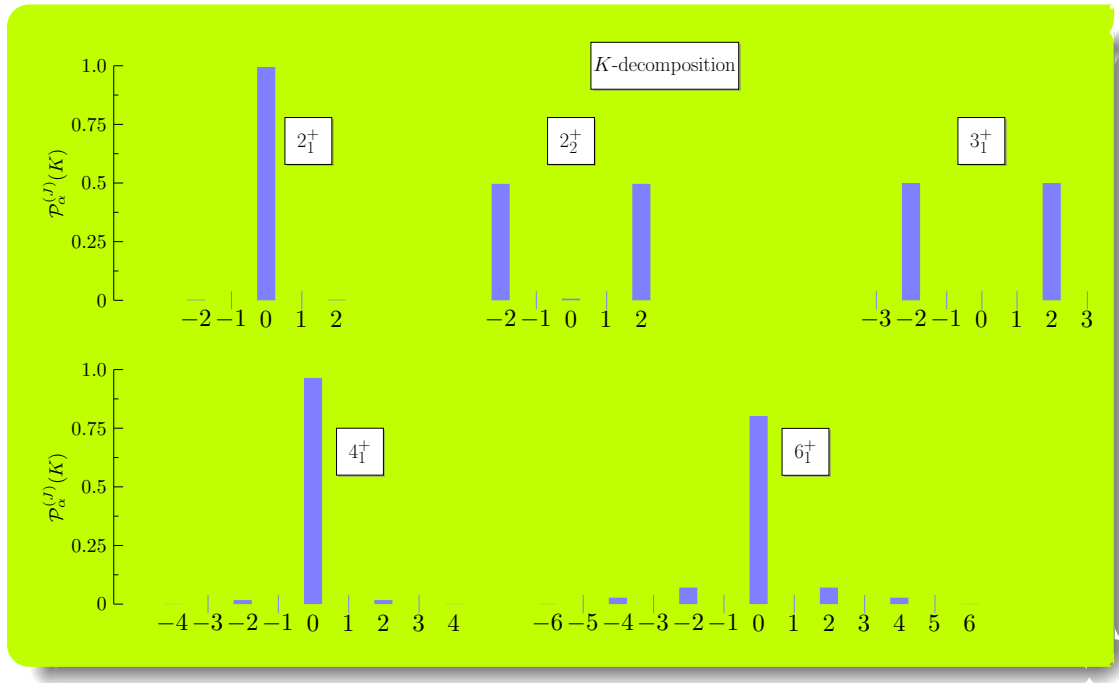
- Rotational symmetry restoration: $\mathcal{P}^J_{MK}|\Phi(q)\rangle \propto |J, M\rangle$

Problem of selecting relevant deformed states: Minimization Technique

"*The first point is the one such that it minimizes the energy. The second point is chosen in such a way that the energy obtained from diagonalizing the Hamiltonian in the 2-dimensional space be a minimum. One proceeds in the same way to determine the third basis vector etc...*" (E. Caurier, Proc. on GCM, BLG report **484** (1975), Bouten, M., Van Leuven, P. (ed.))

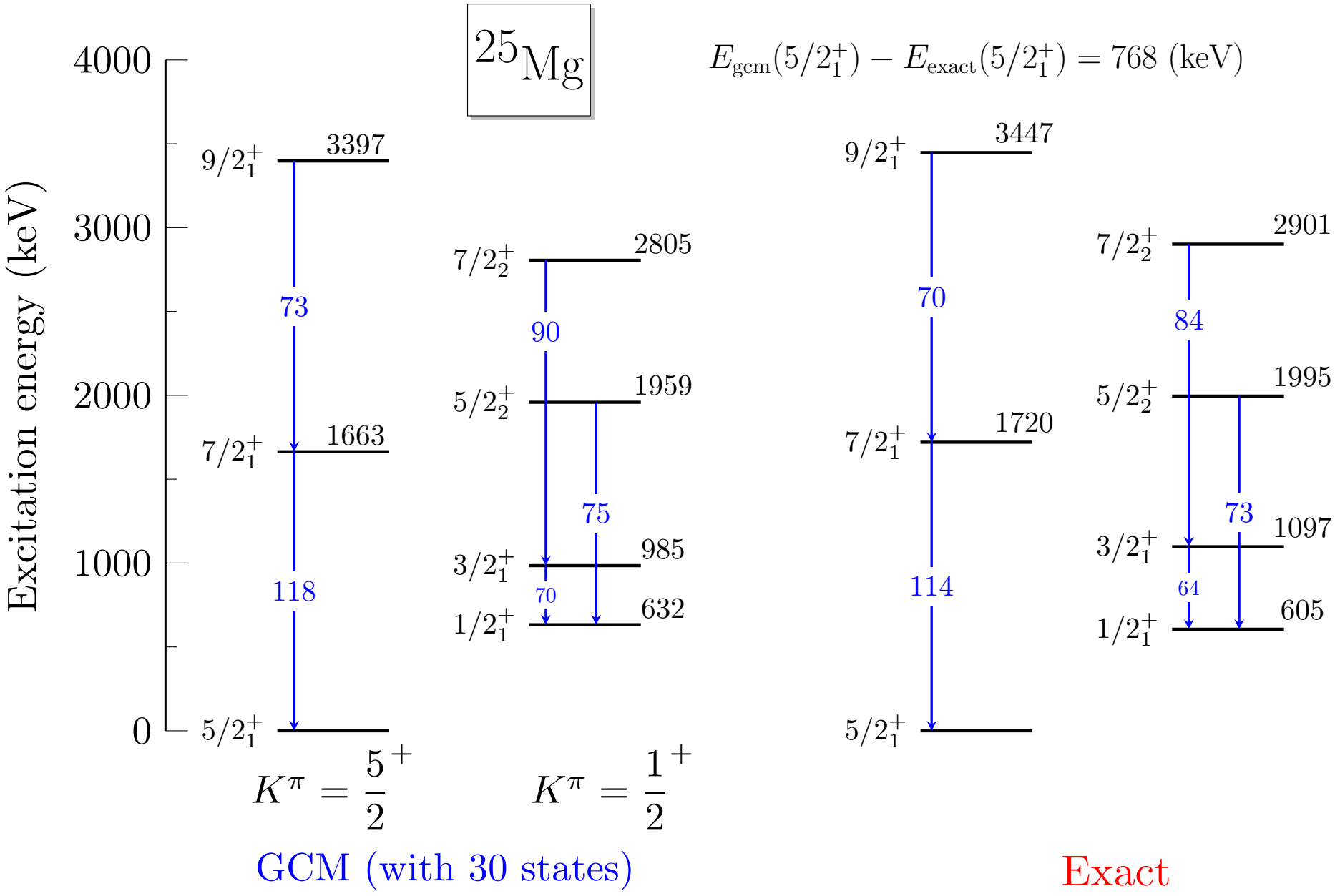
② Systematics in sd nuclei with the USDB interaction





^{24}Mg	$E(2_2^+)$ (keV)	$E(3_1^+)$	$B(E2; 2_2^+ \rightarrow 2_1^+)$ (e^{2}fm^4)	$B(E2; 3_1^+ \rightarrow 2_2^+)$
Exact	4116	5070	14	131
(β, γ)	4059	5140	16	138
$(\beta, \gamma, < \hat{J}_z >)$	4121	5199	17	137

④ Systematics in sd nuclei with the USDB interaction



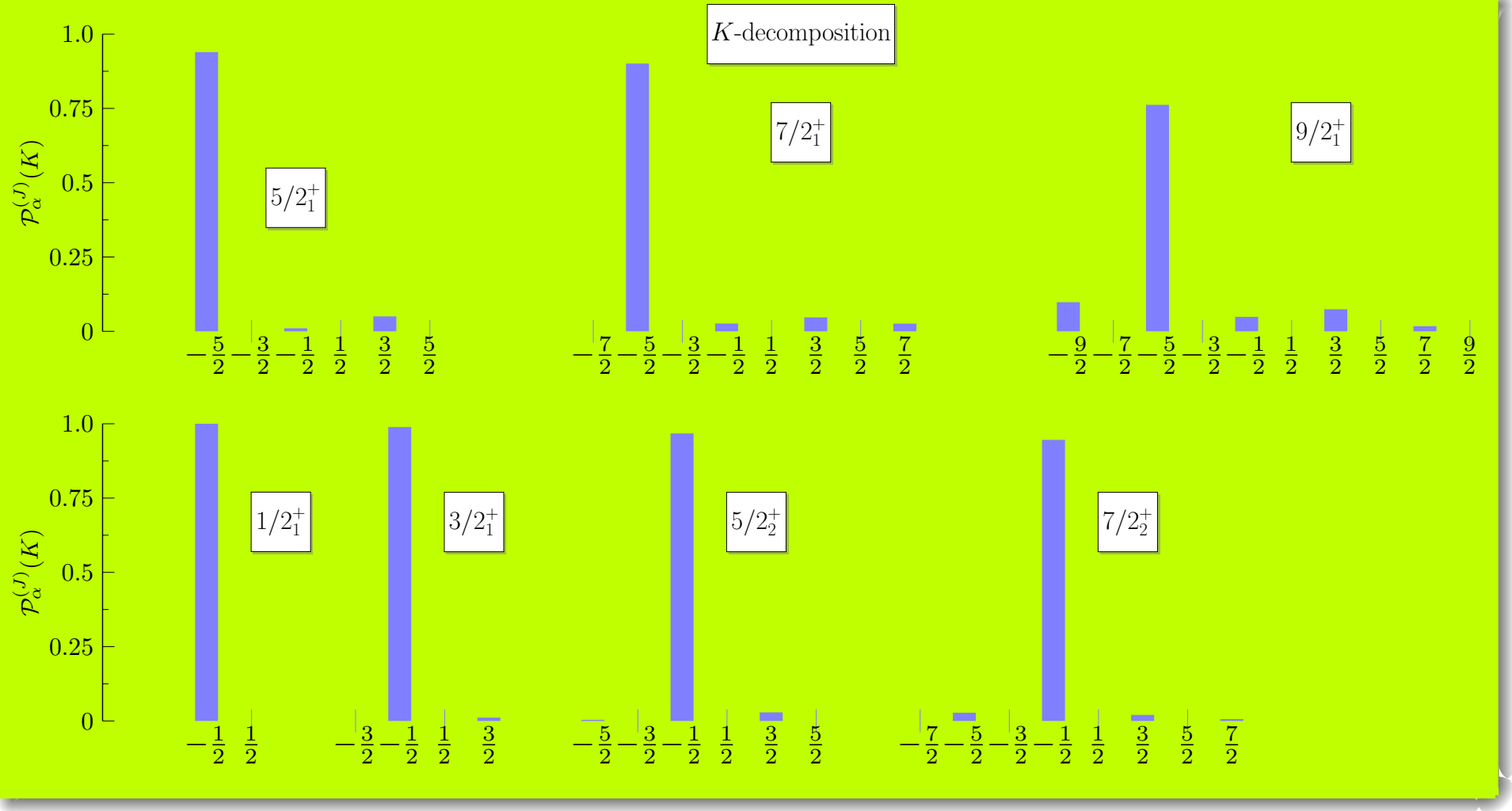
④ Systematics in sd nuclei with the USDB interaction

4000

^{25}Mg

$E_{\rm gcm}(5/2_1^+) - E_{\rm exact}(5/2_1^+) = 768$ (keV)

3447



④ Systematics in sd nuclei with the USDB interaction

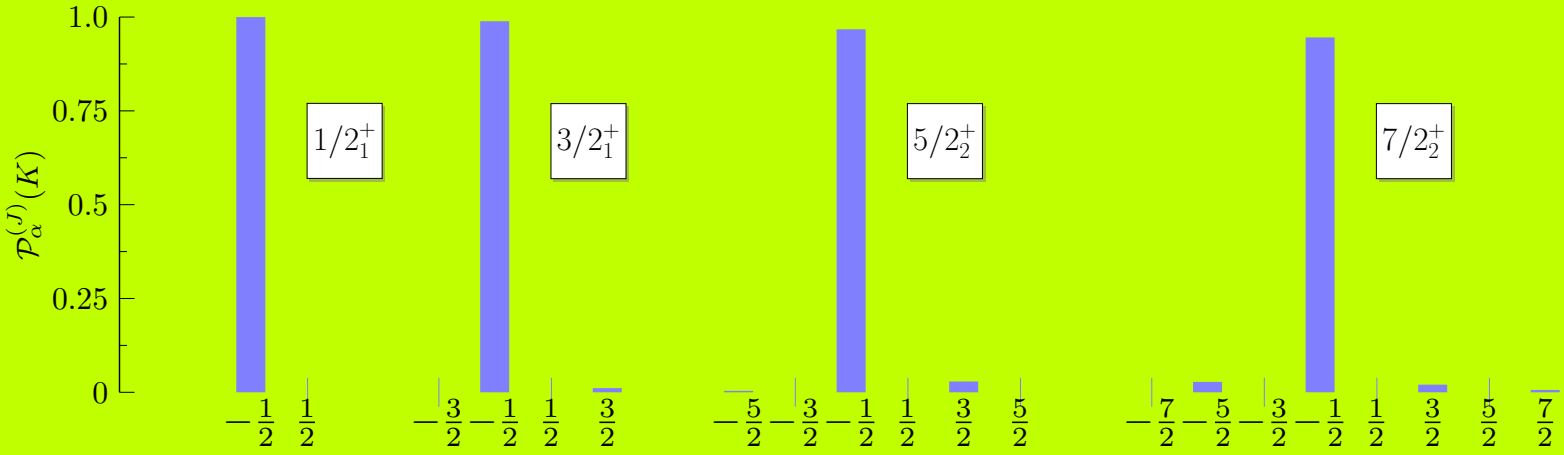
4000

^{25}Mg

$$E_{\mathrm{gcm}}(5/2_1^+) - E_{\mathrm{exact}}(5/2_1^+) = 768\ (\mathrm{keV})$$

Summary for this part:

- Efficient method to choose basis states in the GCM
- Nature of the GCM state: K-decomposition complementary to shell model interpretation in spherical orbits
- Monopole/Multipole decomposition of the effective Hamiltonian



PHYSICAL REVIEW LETTERS **124,** 152501 (2020)

Establishing the Maximum Collectivity in Highly Deformed $N=Z$ Nuclei

R. D. O. Llewellyn,[1,*] M. A. Bentley,[1] R. Wadsworth,[1,†] H. Iwasaki,[2,3] J. Dobaczewski,[1,4] G. de Angelis,[5] J. Ash,[2,3] D. Bazin,[2,3] P. C. Bender,[2,‡] B. Cederwall,[6] B. P. Crider,[2,§] M. Doncel,[7] R. Elder,[2,3] B. Elman,[2,3] A. Gade,[2,3] M. Grinder,[2,3] T. Haylett,[1] D. G. Jenkins,[1] I. Y. Lee,[8] B. Longfellow,[2,3] E. Lunderberg,[2,3] T. Mijatović,[2,‖] S. A. Milne,[1] D. Muir,[1] A. Pastore,[1] D. Rhodes.[2,3] and D. Weisshaar[2]

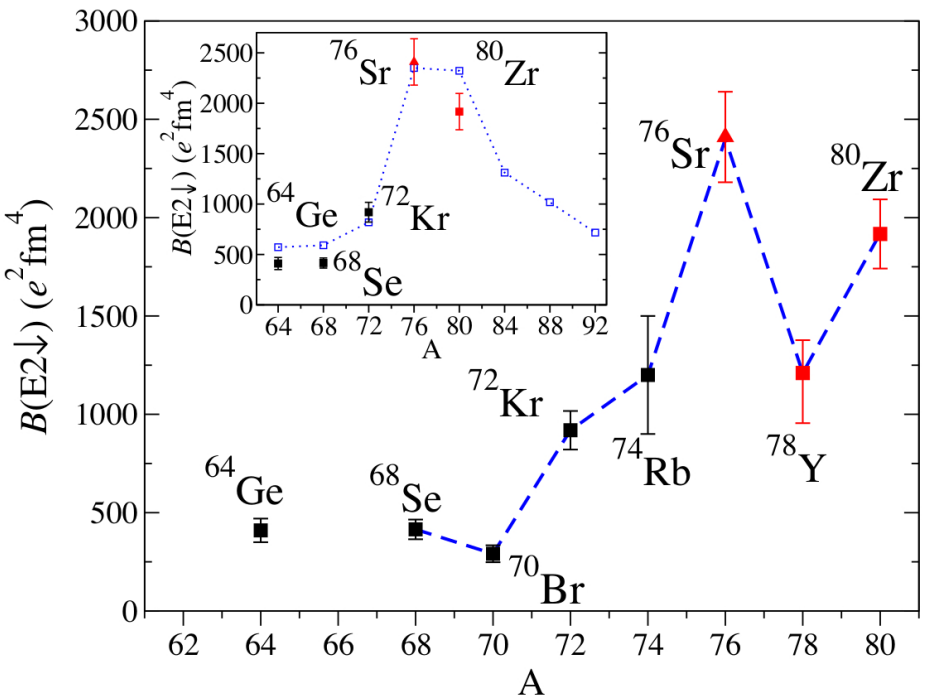
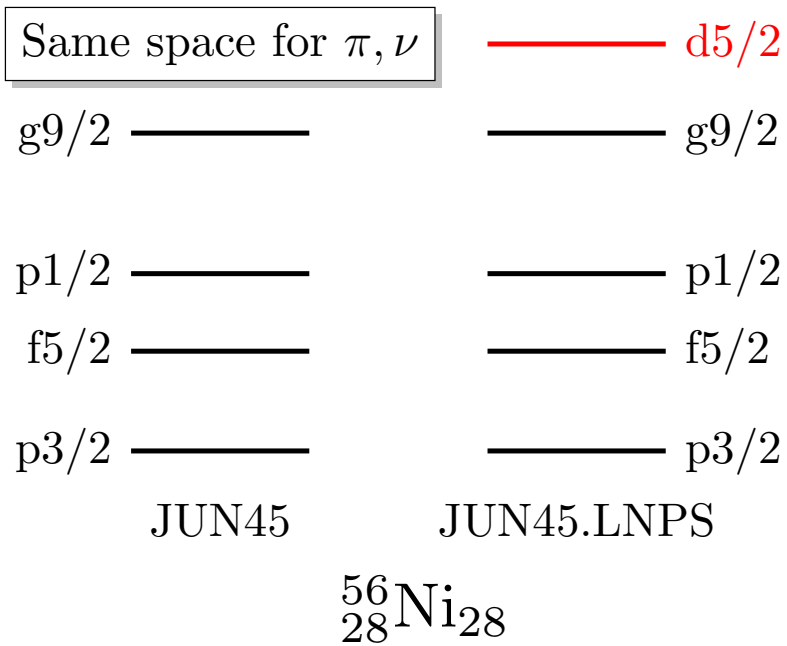
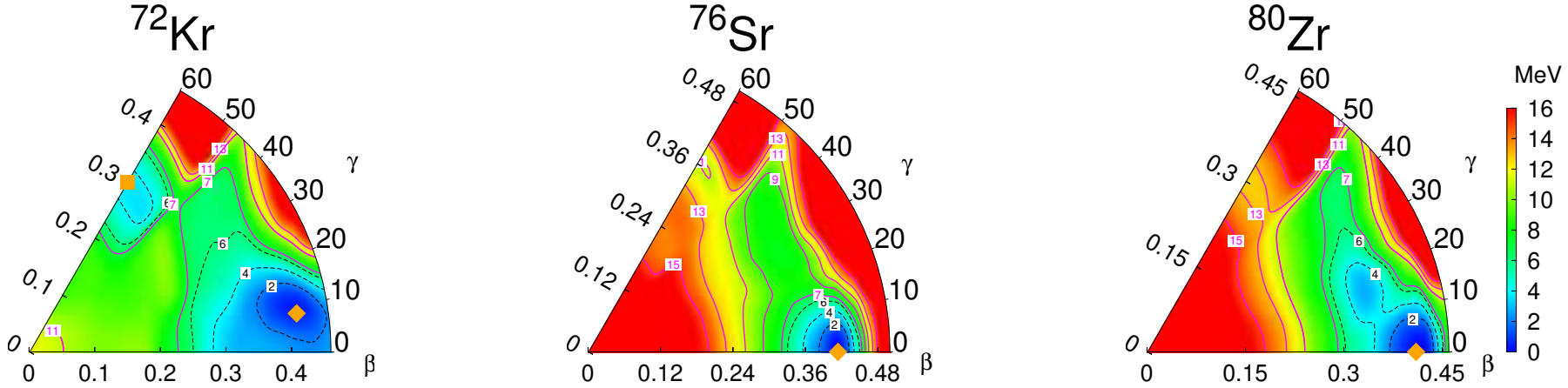


FIG. 3. Schematics of the $B(E2\downarrow)$ values for the $N = Z$ nuclei

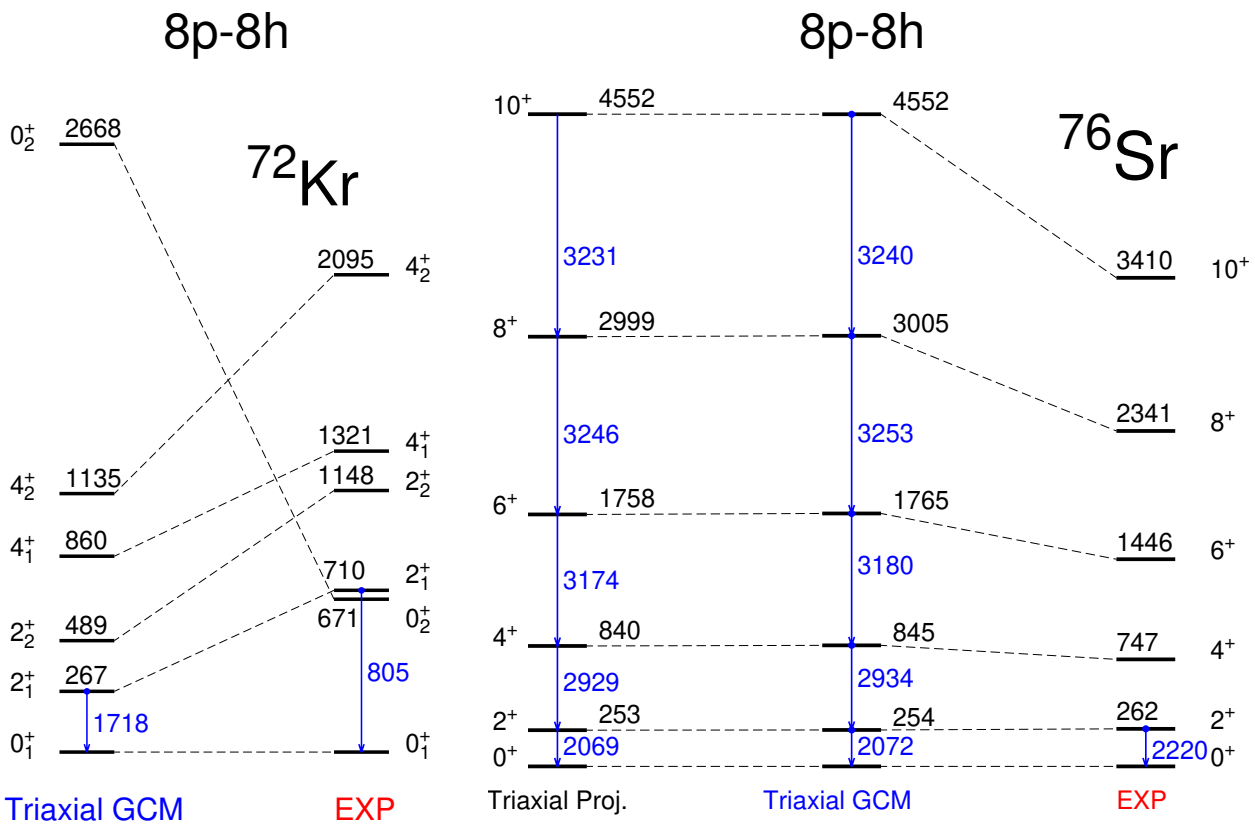
- Presence of d5/2 $\Longrightarrow$ deformation
- LNPS matrix elements

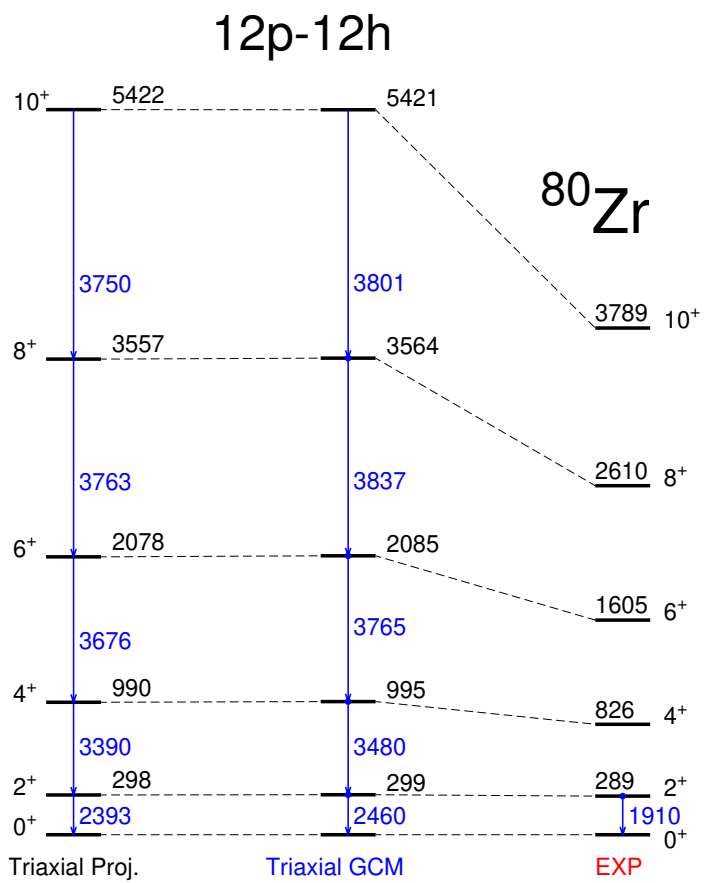


6 Collectivity in heavy $N = Z$ nuclei



- HF minimum and ground state structures:





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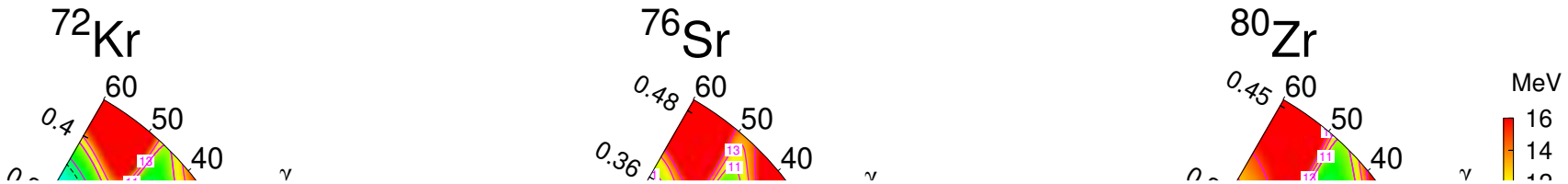
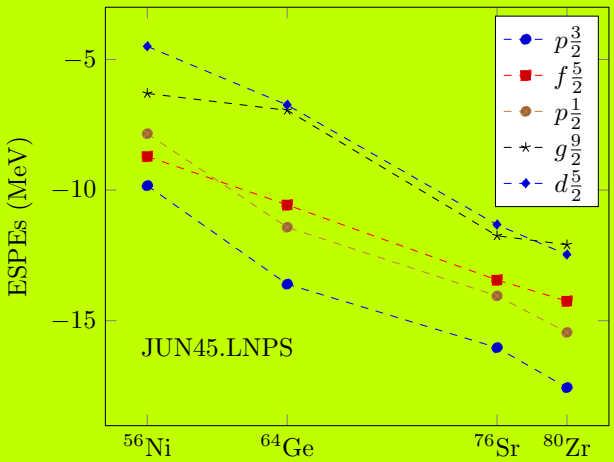
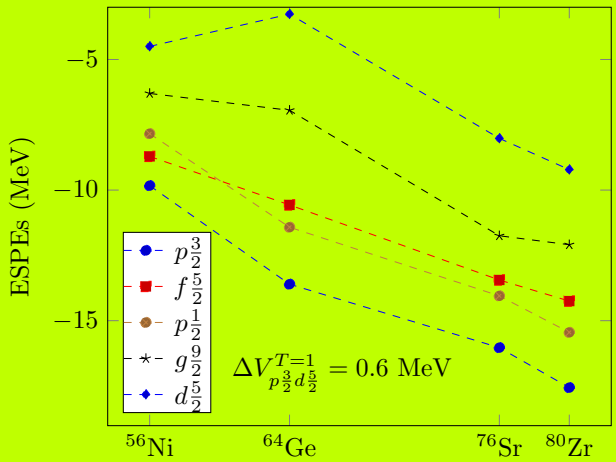
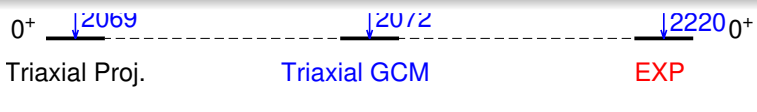


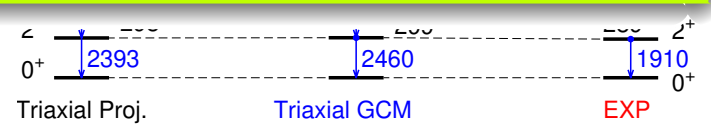
- Effective Single Particle Energies (ESPEs)





- Summary for this part:
  - First calculations of heavy $N = Z$ nuclei within the Shell Model
  - Good description in the case of ^{76}Sr, ^{80}Zr
  - Work under progress to determine the location of the $d\frac{5}{2}$ orbit (systematics of second excited states 0^+_2 where JUN45 becomes inadapted)





⑦ $^{254}_{102}No_{152}$: Preliminary calculations with the GCM

Kuo-Herling interaction:

- $^{208}_{82}Pb_{126}$ core, realistic TBMEs
- $82 \leq Z \leq 126$ shells for proton and $126 \leq N \leq 184$ for neutrons
- monopole corrections (3N force)

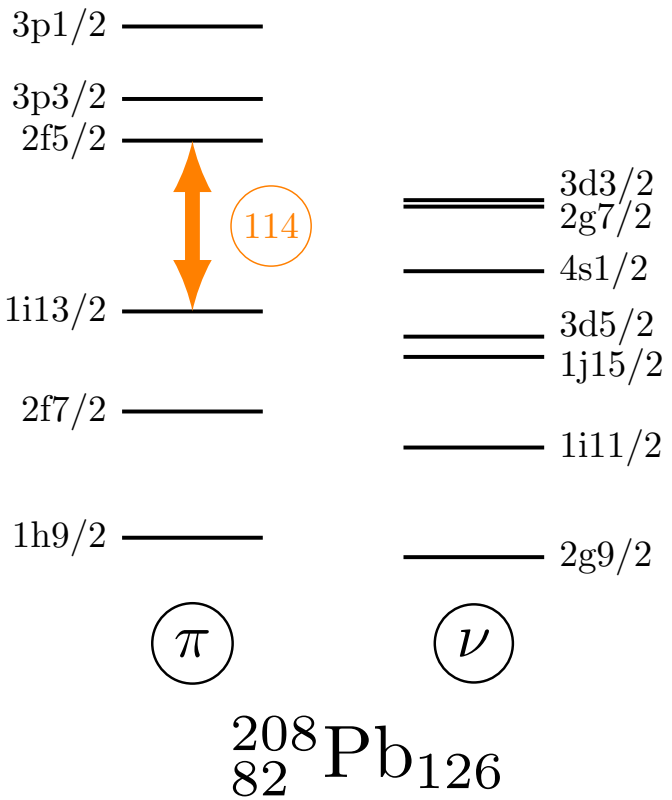
E. Caurier and F. Nowacki,

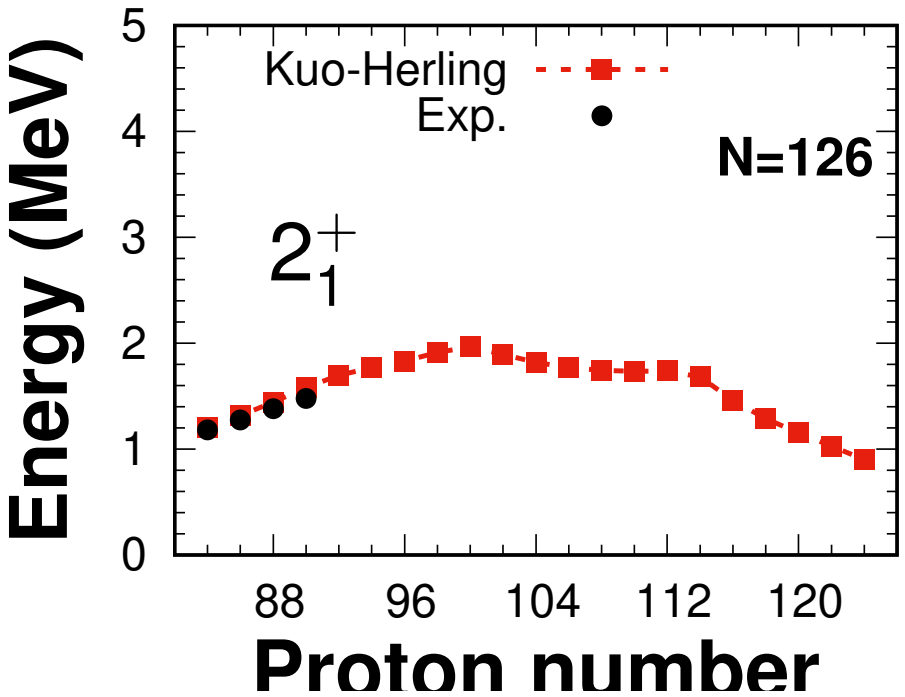
*PRL **87** (2001),072511*

Calculations: NATHAN & DFSM

◇ Diagonalization within the seniority scheme along the chains of $N = 126$ and $N = 184$

◇ Deformed Hartree-Fock and Angular momentum projection plus shapes mixing through the GCM: ^{254}No





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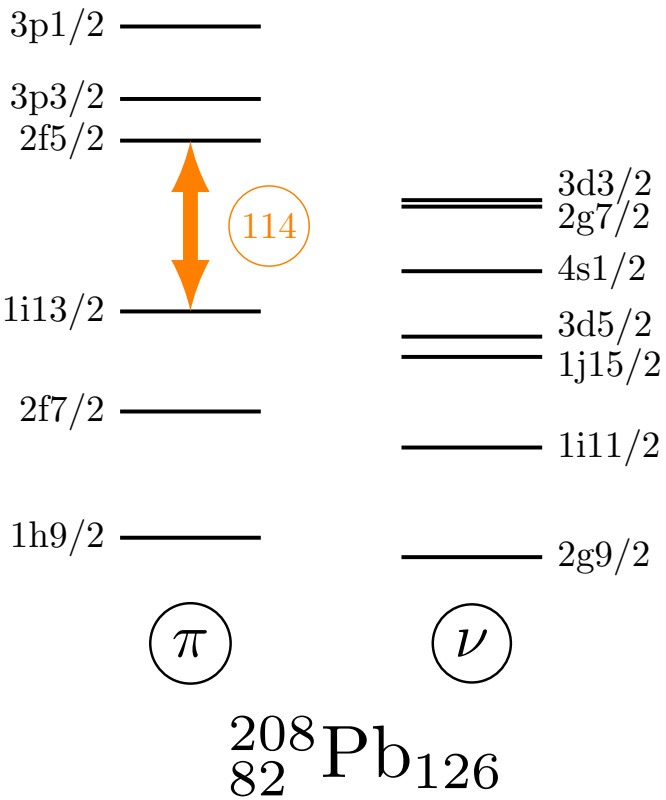

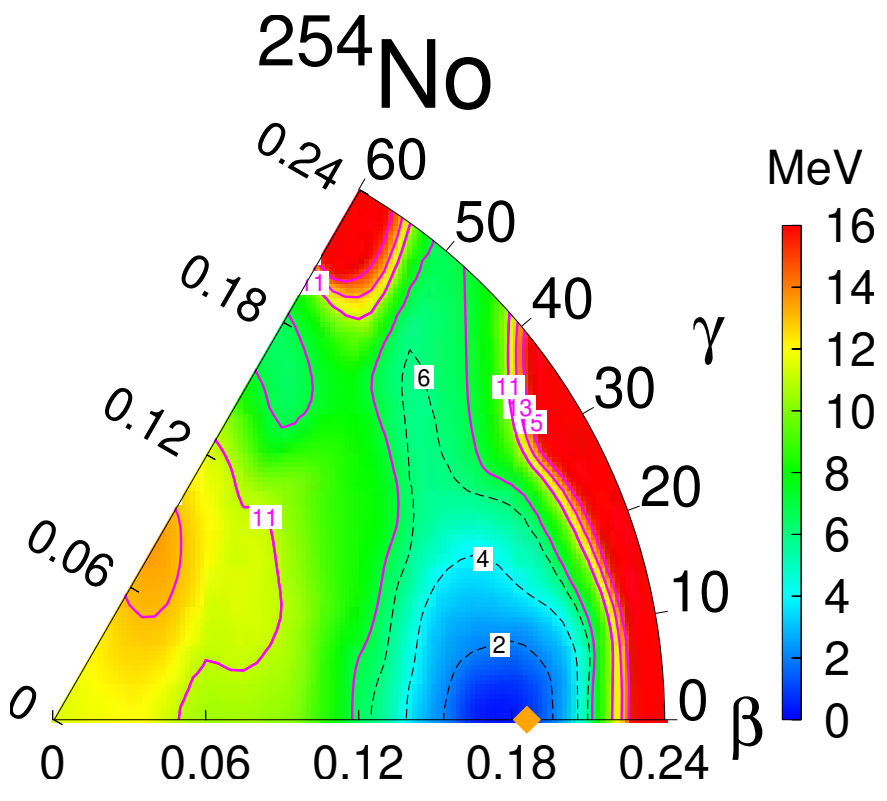

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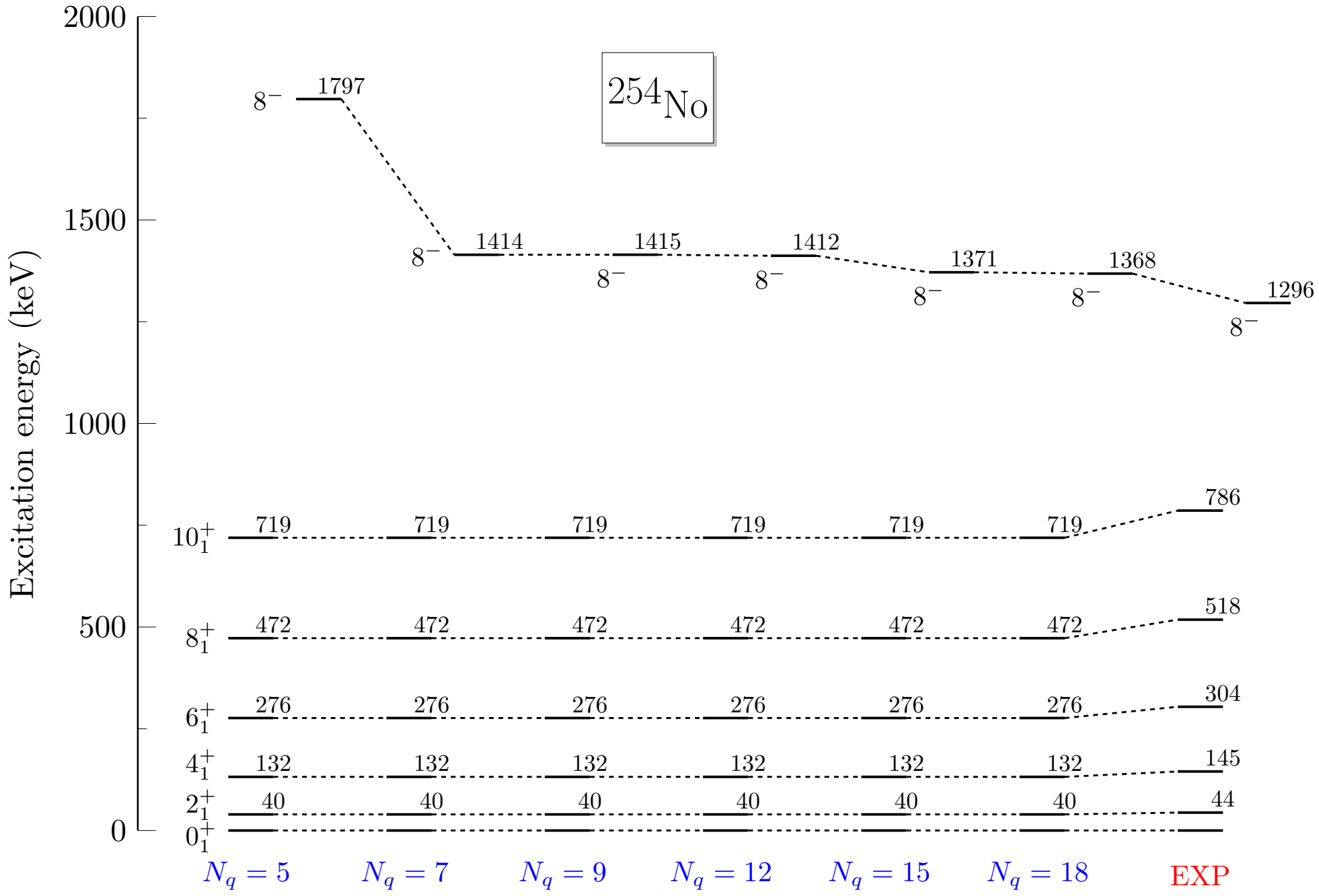

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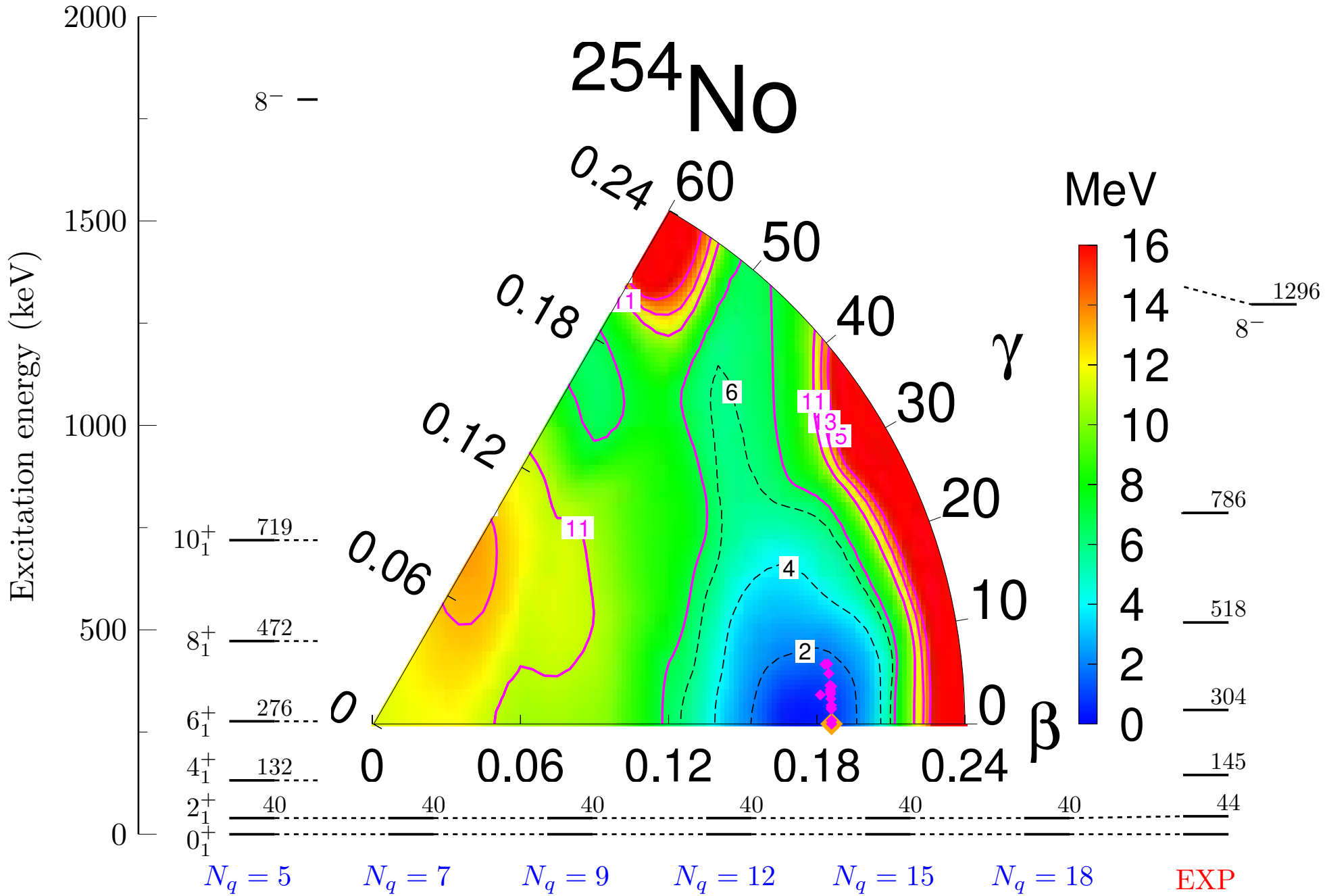

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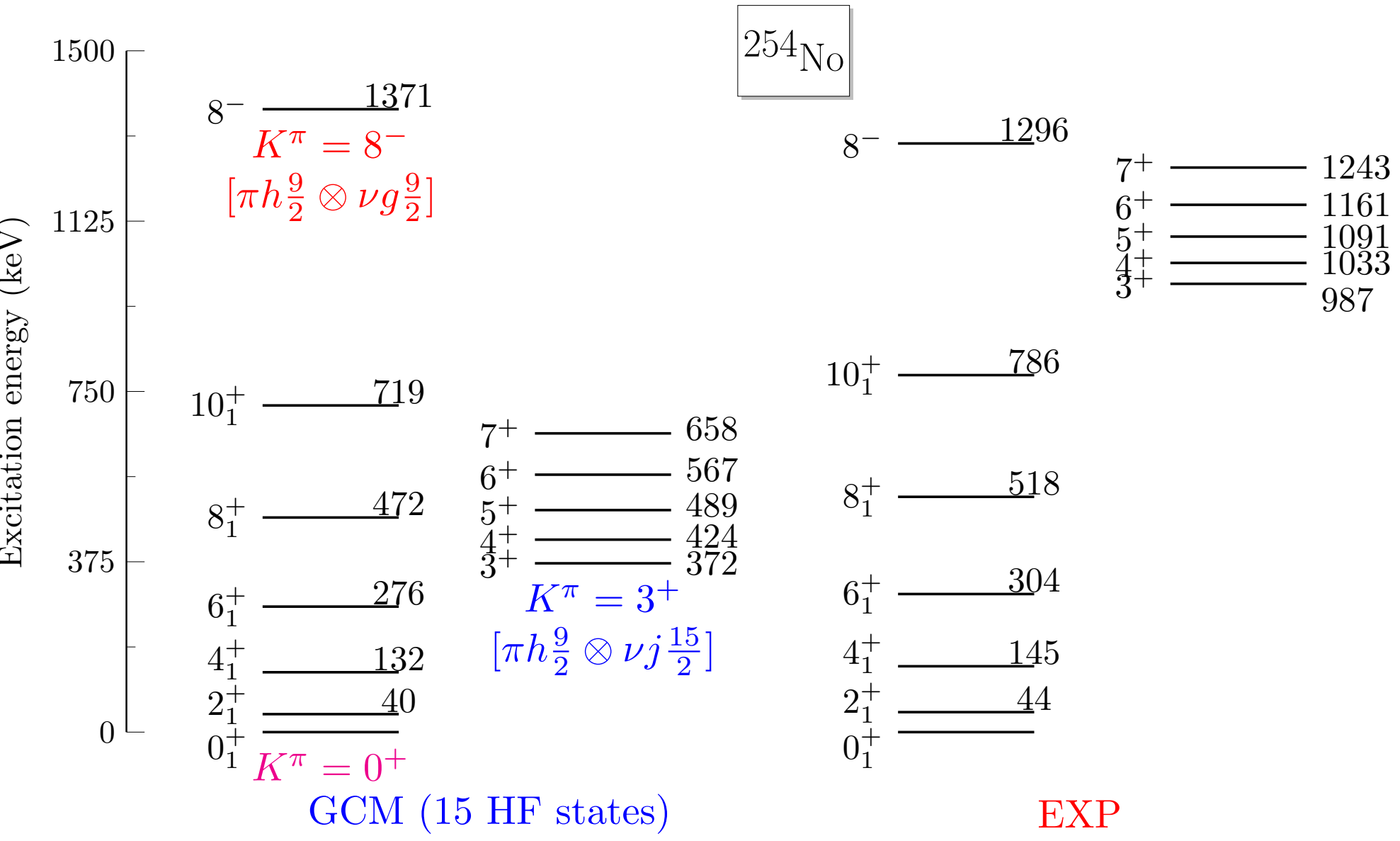

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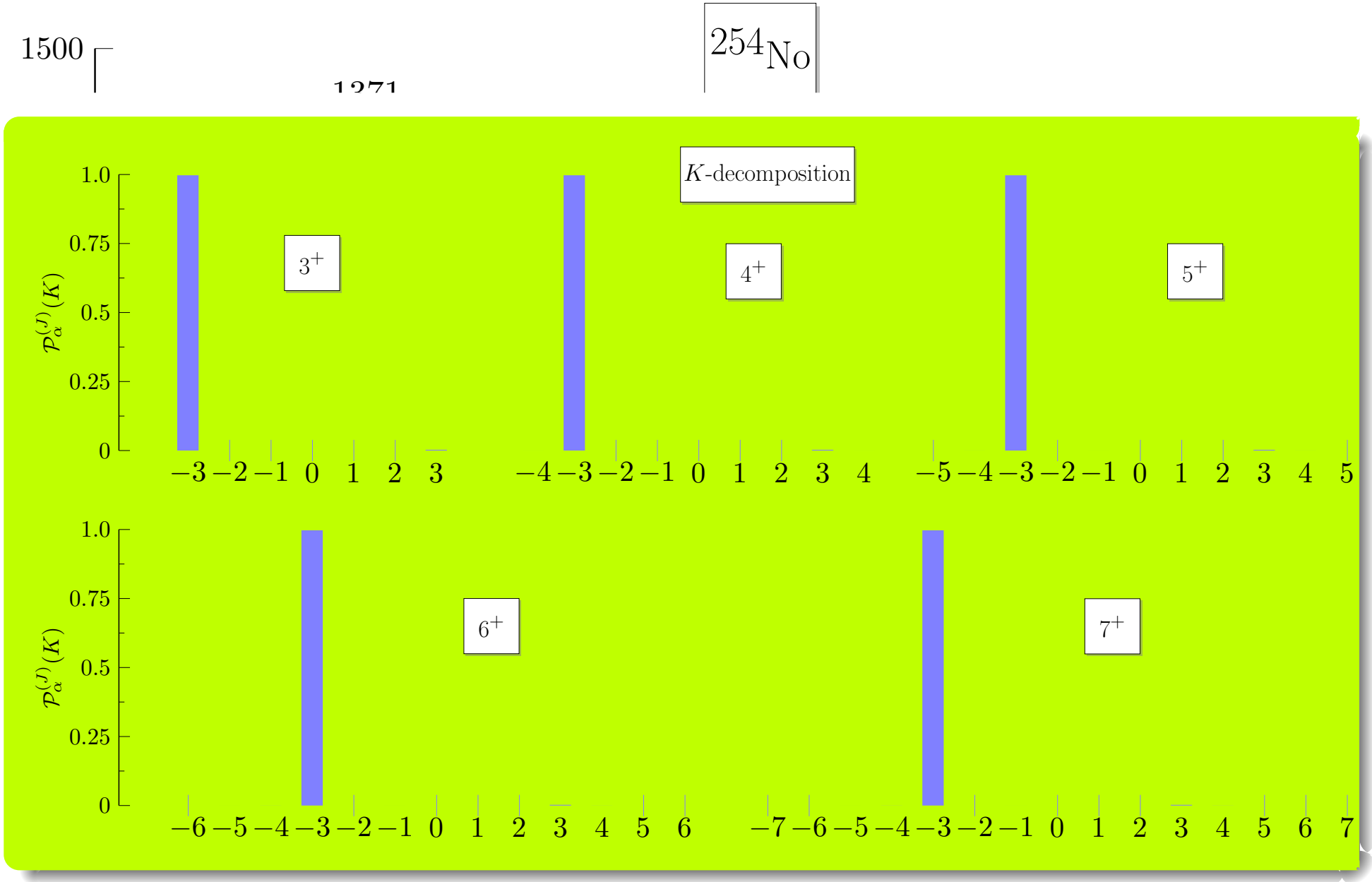

Occupation numbers:

proton orbits	$1h_{9/2}$	$1i_{13/2}$	$2f_{7/2}$	$2f_{5/2}$	$3p_{3/2}$	$3p_{1/2}$	
0_1^+	5.66	7.99	3.44	1.58	0.76	0.57	
8^-	6.52	7.82	3.28	1.20	0.79	0.39	
3^+	6.50	7.98	3.31	1.14	0.72	0.35	
neutron orbits	$1i_{11/2}$	$1j_{15/2}$	$2g_{9/2}$	$2g_{7/2}$	$3d_{5/2}$	$3d_{3/2}$	$4s_{1/2}$
0_1^+	7.28	9.67	5.45	1.11	1.16	0.87	0.46
8^-	7.29	9.04	6.07	1.12	1.15	0.88	0.45
3^+	7.31	9.94	5.43	0.99	1.07	0.83	0.43

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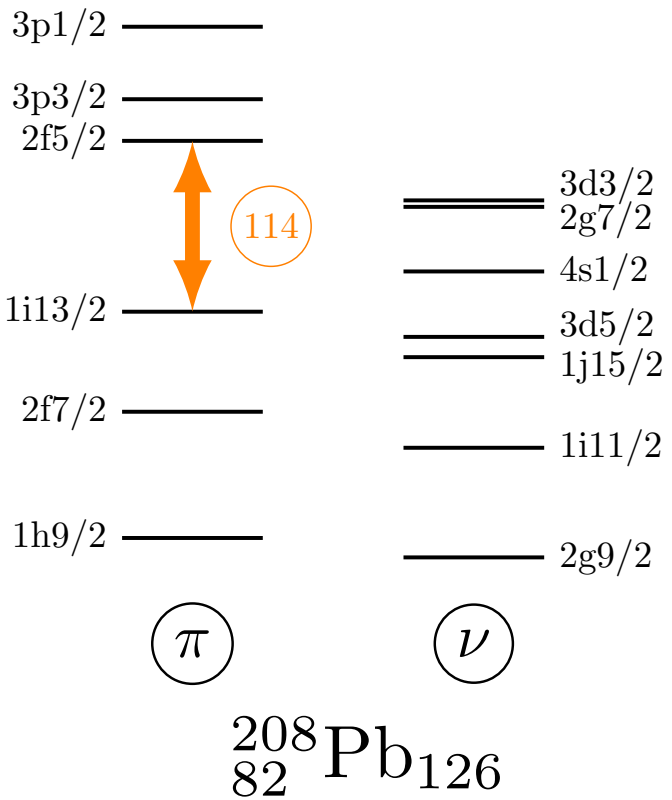
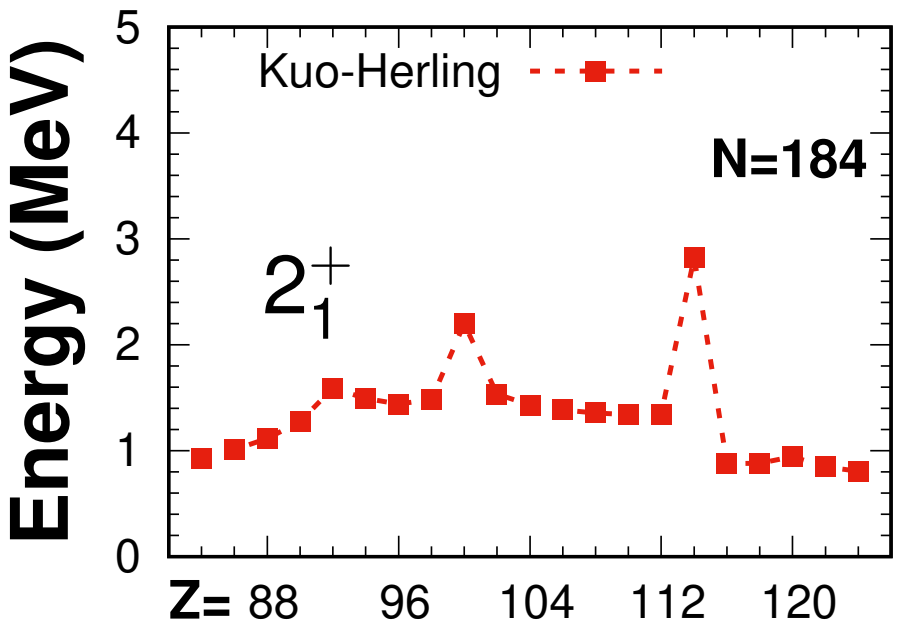




⑦ $^{254}_{102}No_{152}$: Preliminary calculations with the GCM

Kuo-Herling interaction:

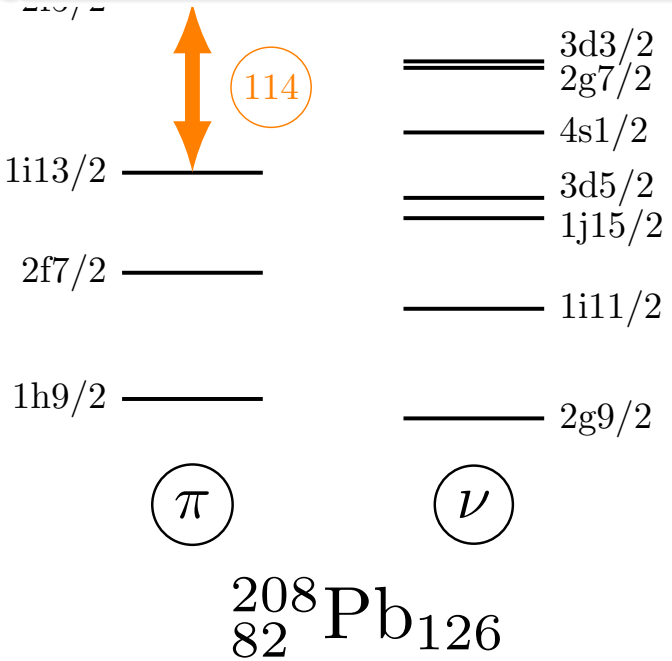
- $^{208}_{82}Pb_{126}$ core, realistic TBMEs

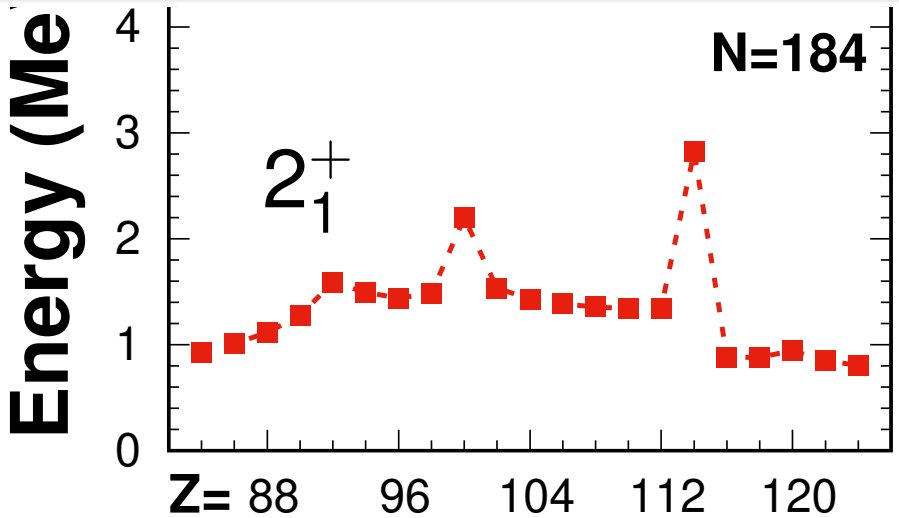
Calculations: NATHAN & DFSM

- Diagonalization within the seniority

Summary for this part:

- First Shell Model interpretation of the superheavy element ^{254}No
- Simultaneous description of the rotational band AND the isomeric states 8^- and the 3^+ band
- Stability predicted at $Z = 114$

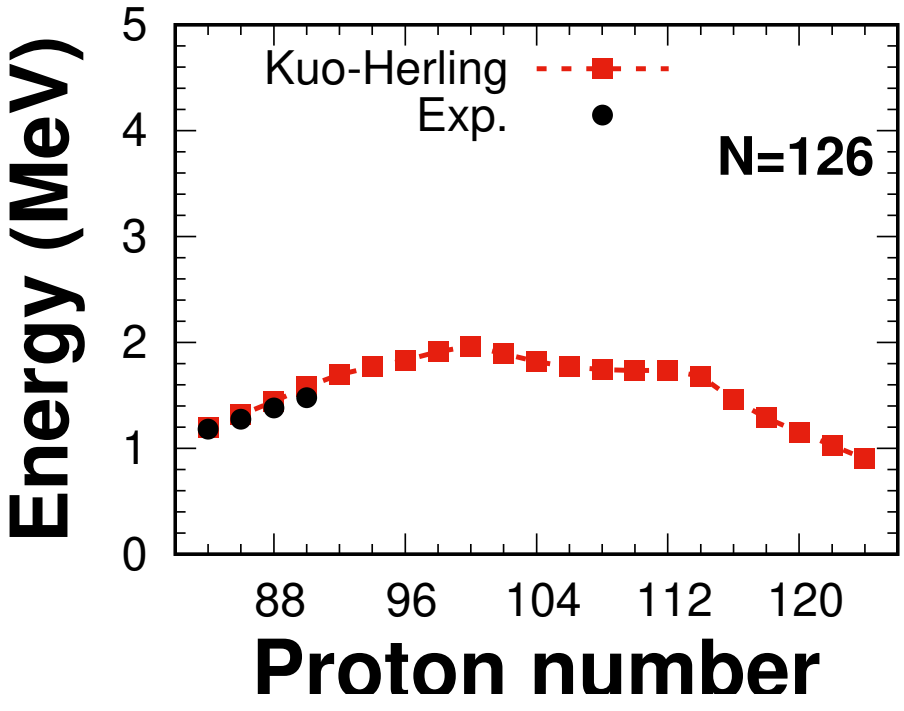


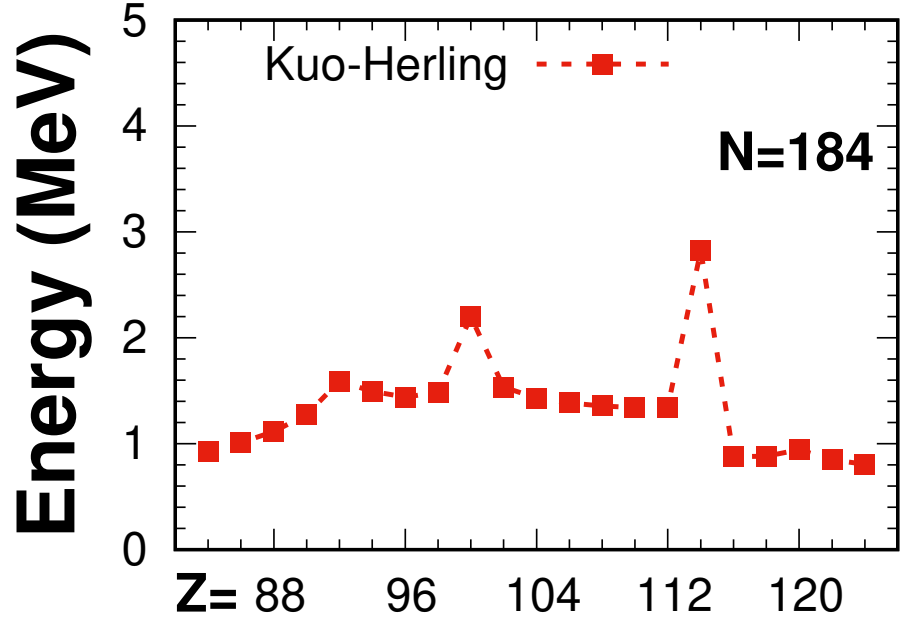


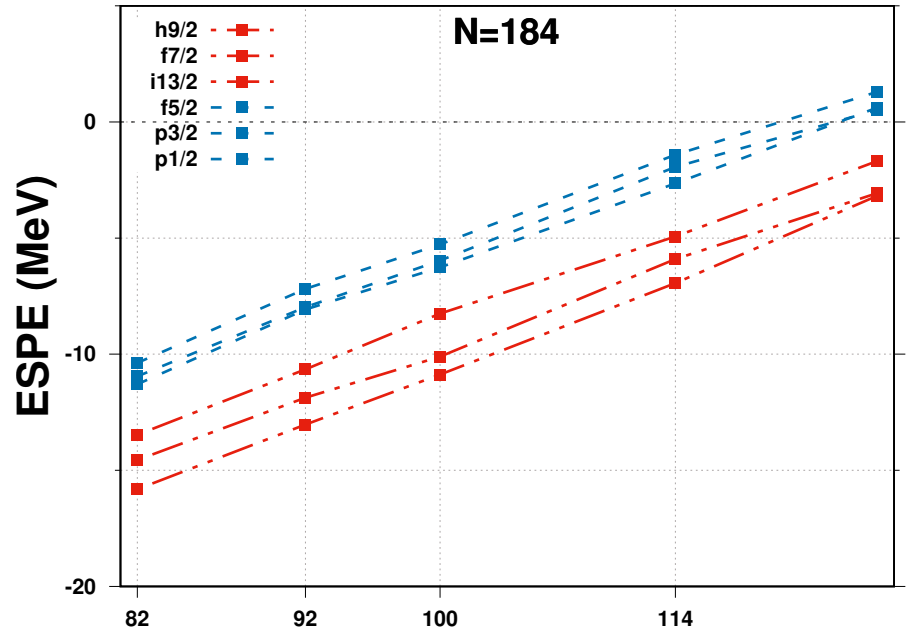
Summary

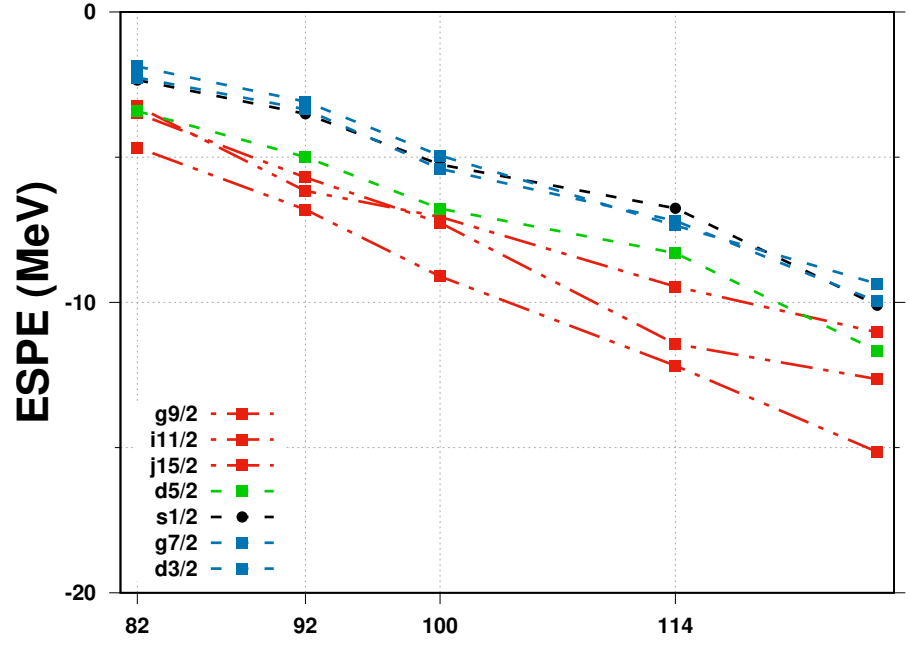
- Shell model framework:
  Diagonalization versus Generator Coordinate Method (GCM)
- Systematics calculations of sd shell nuclei
- Collectivity in heavy $N = Z$ nuclei
  with the effective JUN45.LNPS interaction
- ^{254}No: Preliminary calculations and comparison with experimental data (M. Forge talk on Monday)

BACK UP

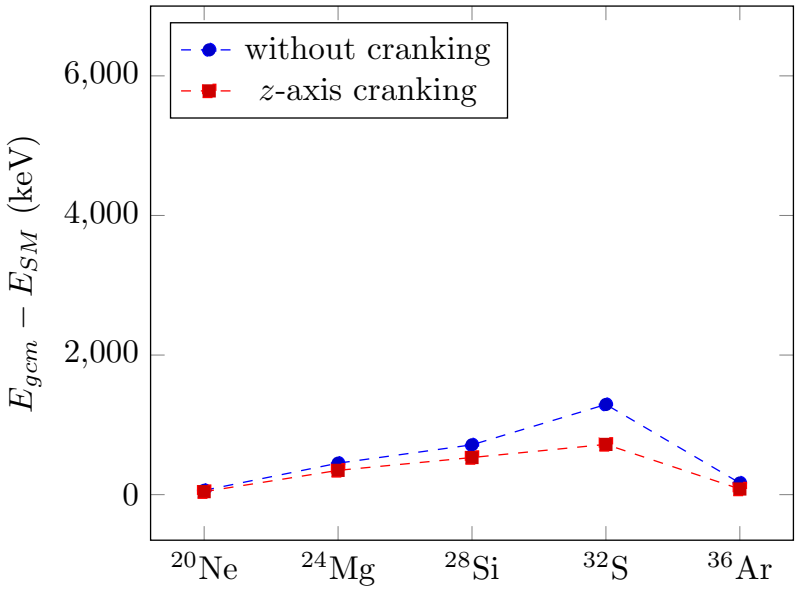

Kuo-Herling
Exp.
N=126
Energy (MeV)
Proton number
0
1
2
3
4
5
88
96
104
112
120

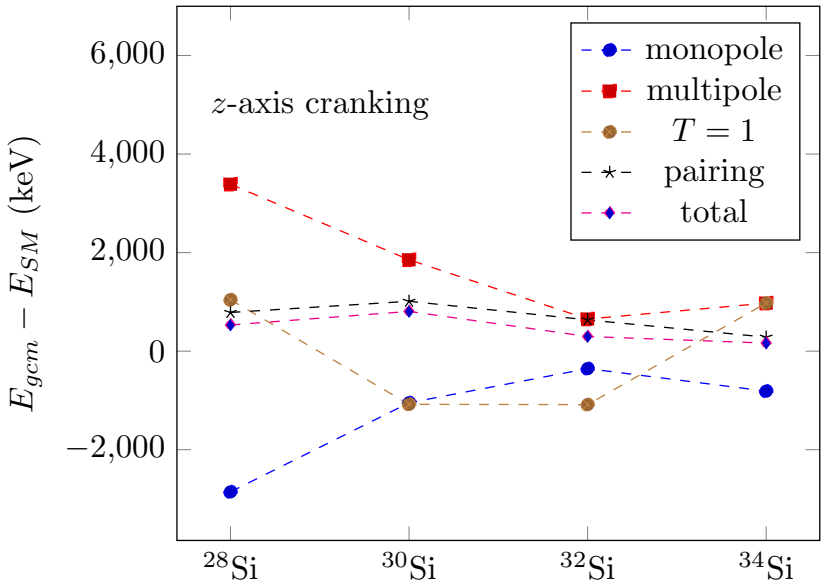

Kuo-Herling
N=184
Energy (MeV)
Z=
0
1
2
3
4
5
88
96
104
112
120

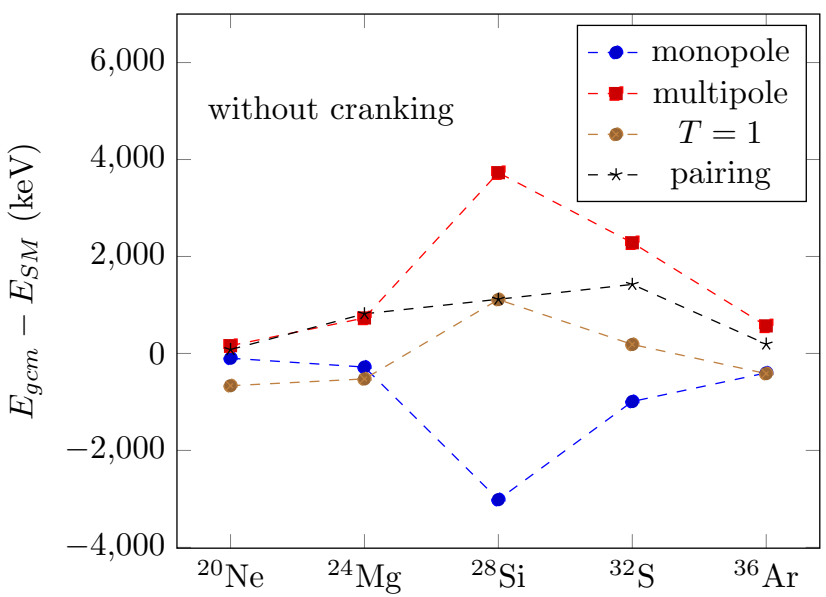

N=184
h9/2
f7/2
i13/2
f5/2
p3/2
p1/2
ESPE (MeV)
0
-10
-20
82
92
100
114

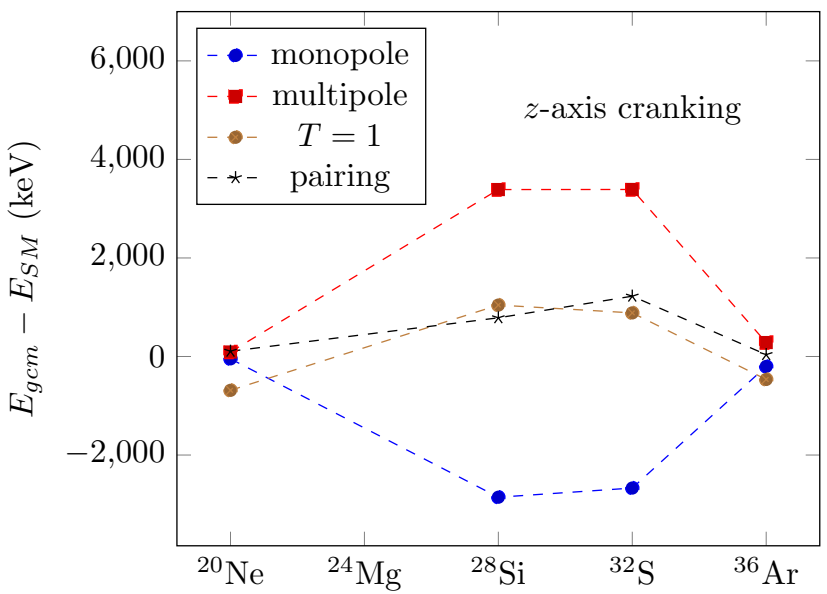

g9/2
i11/2
j15/2
d5/2
s1/2
g7/2
d3/2
ESPE (MeV)
0
-10
-20
82
92
100
114

Systematics in sd nuclei with the USDB interaction

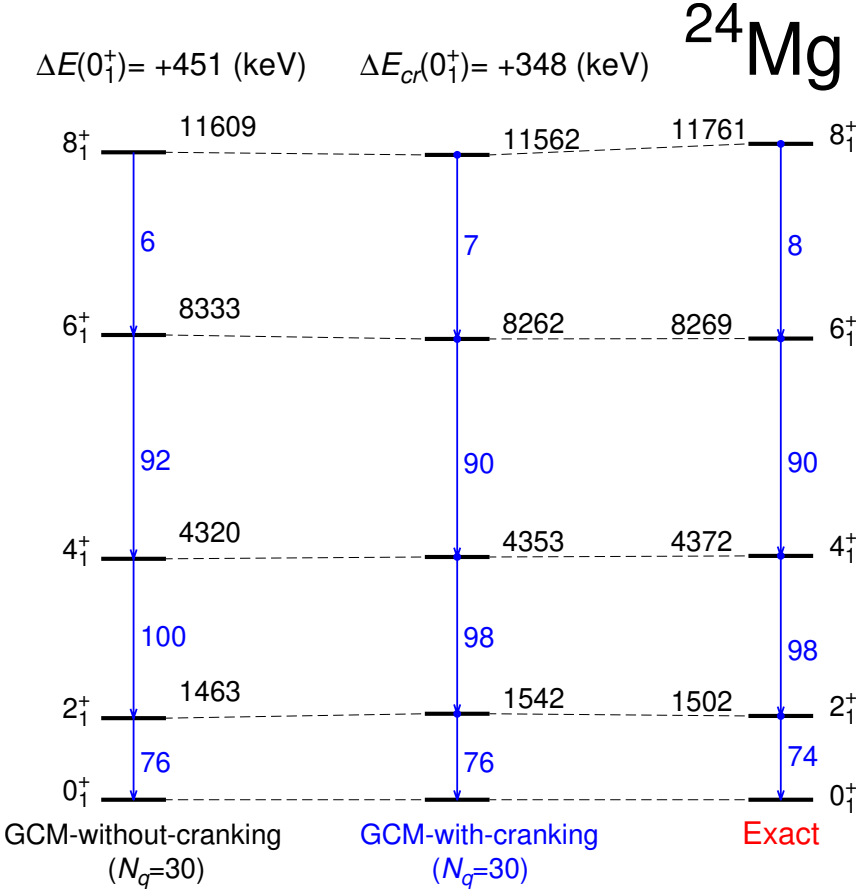


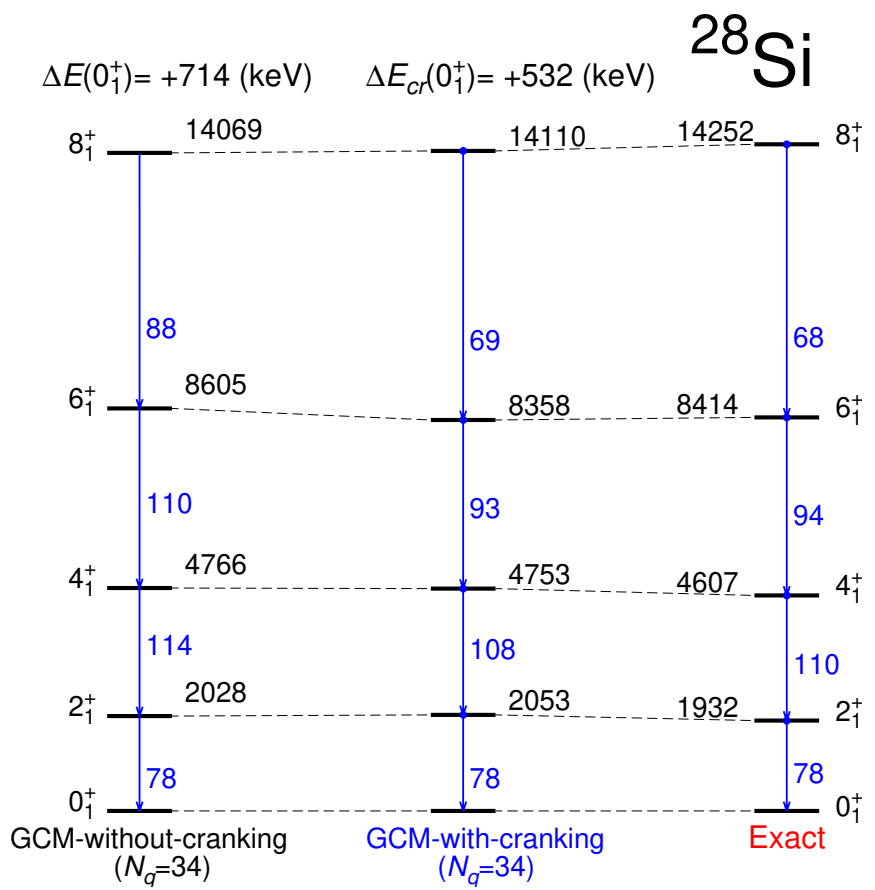






② Systematics in sd nuclei with the USDB interaction





^{24}Mg	$E(2_2^+)$ (keV)	$E(3_1^+)$	$B(E2; 2_2^+ \to 2_1^+)$ (e^{2}fm^4)	$B(E2; 3_1^+ \to 2_2^+)$
Exact	4116	5070	14	131
(β, γ)	4059	5140	16	138
$(\beta, \gamma, < \hat{J}_z >)$	4121	5199	17	137

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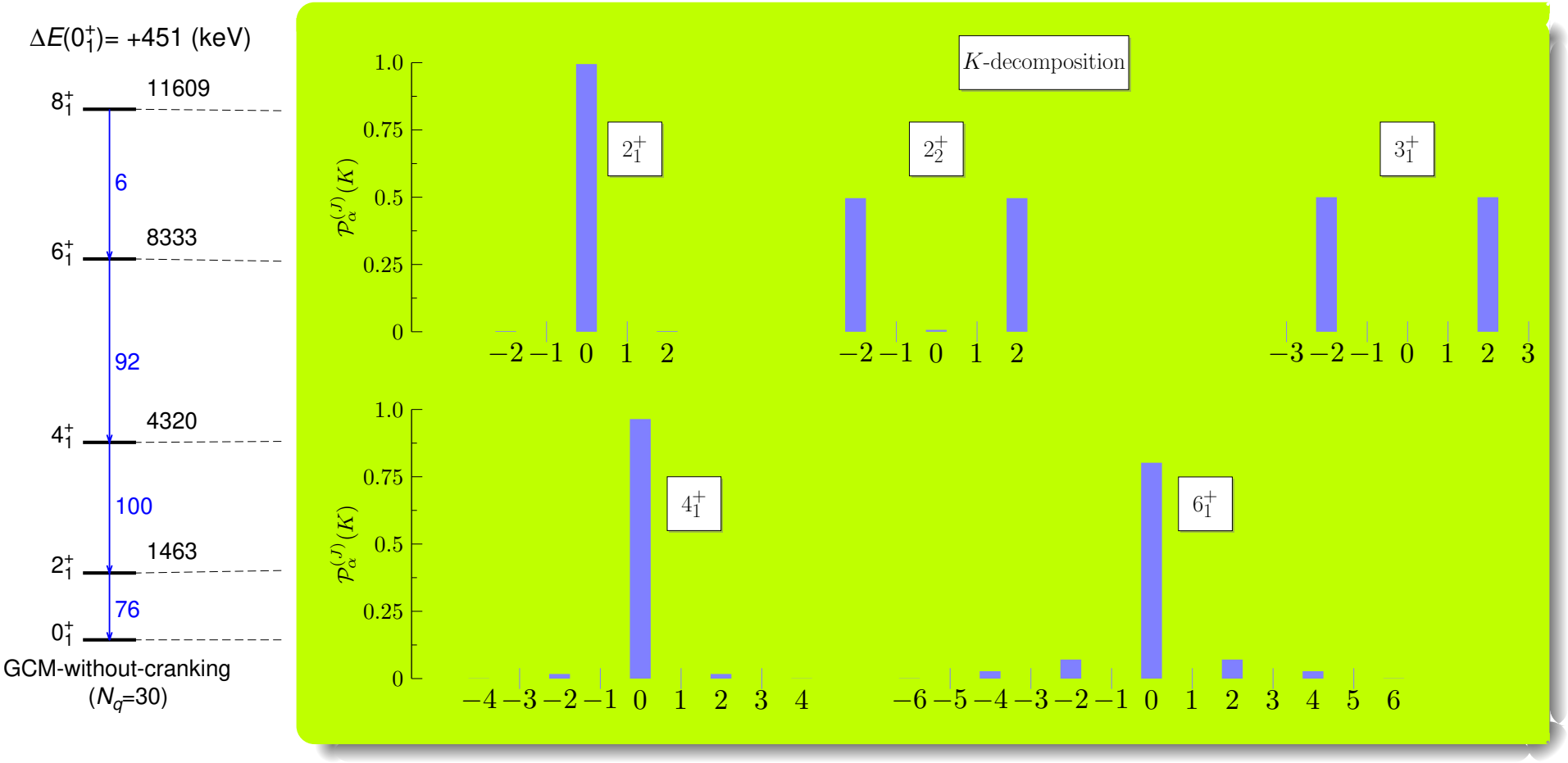

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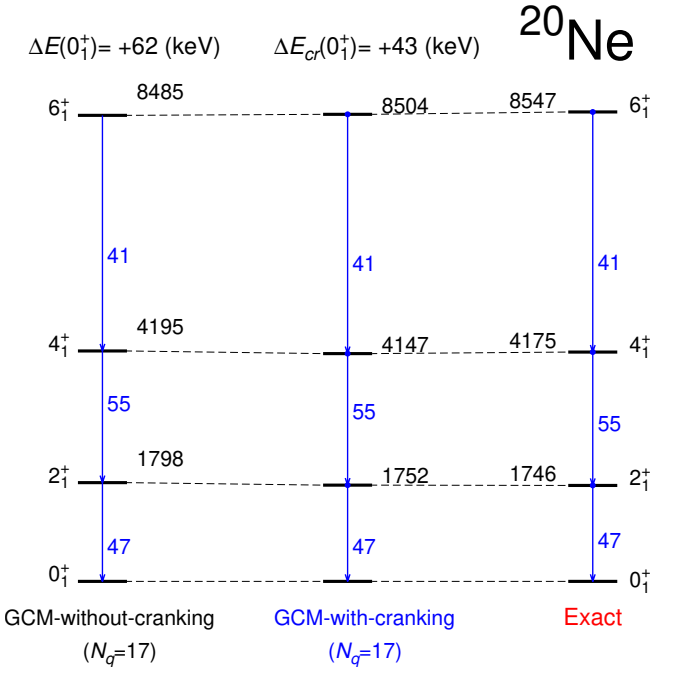

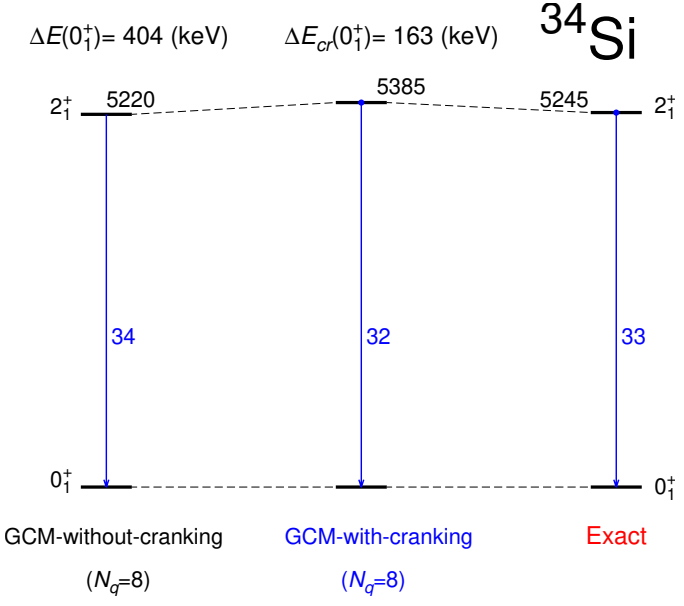

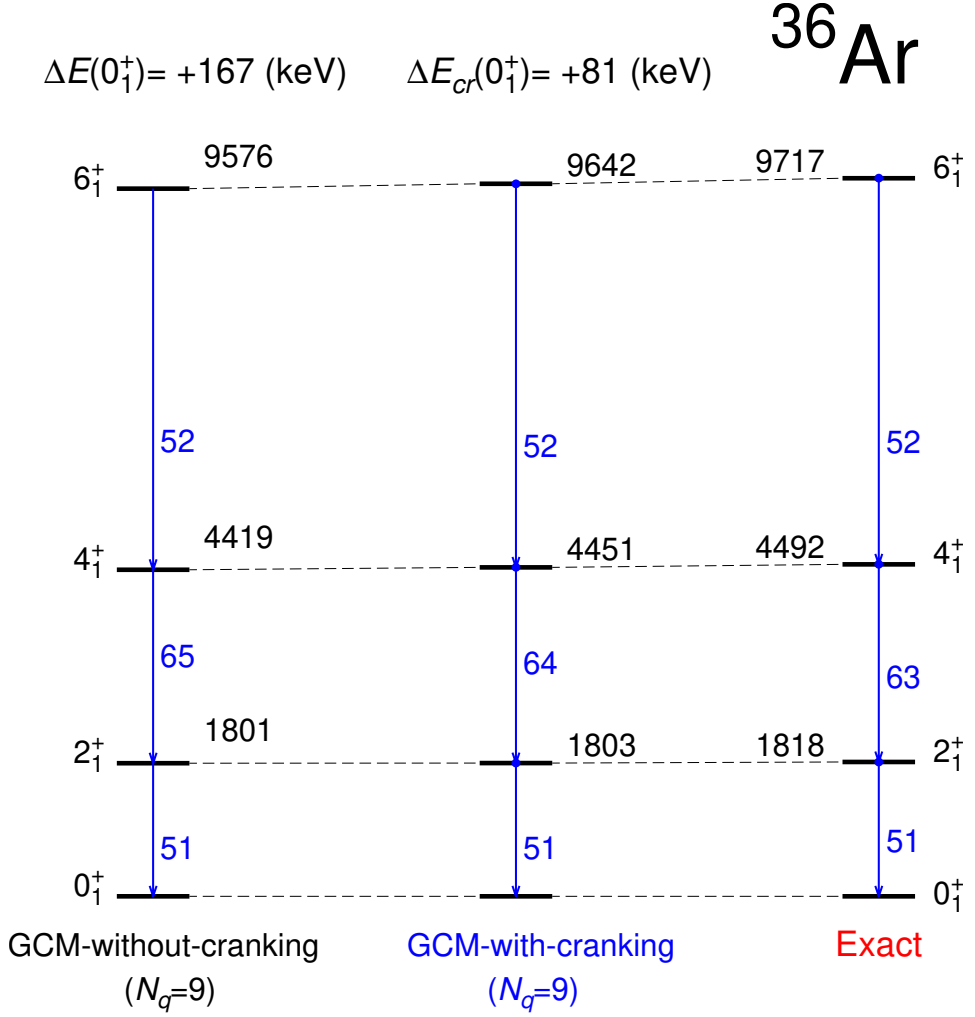

- Different deformation regimes can be treated with the minimization technique;
- Missing energy of the ground state largest at the middle of the shell $\sim 350 - 700$ (keV)