Theoretical and experimental challenges in isospin symmetry breaking studies

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Outline

Isospin symmetry breaking in
Energy differences between mirror nuclei (MED)
• How we calculate them within the shell model
• The role of the Coulomb force
• Other isospin symmetry breaking terms
• Systematic study of MED
• Application to nuclei in the pf+gds shells: The A=73 case

Isospin symmetry breaking in
Electromagnetic transition probabilities
• Recent results with stable and radioactive beams
• Application to nuclei in the pf+gd shells: The A=70 case
Neutron-proton exchange symmetry

Charge symmetry: $V_{pp} = V_{nn}$

Charge independence: $(V_{pp} + V_{nn})/2 = V_{np}$

Deviations are small

The electromagnetic interaction lifts the degeneracy of the analogue states but does not, in general, affect the underlying symmetry
Differences in analogue excited states

Mirror Energy Differences (MED)

\[ \text{MED}_J = \text{Ex}_{J,T_z=-T} - \text{Ex}_{J,T_z=+T} \]

Test the charge symmetry of the interaction

Triplet Energy Differences (TED)

\[ \text{TED}_J = \text{Ex}_{J,T_z=-T} + \text{Ex}_{J,T_z=+T} - 2\text{Ex}_{J,T_z=0} \]

Test the charge independency of the interaction
Mirror energy differences

difference in excitation energies

\[
\text{MED}_J = E^*_{J,-|T_z|} - E^*_{J,|T_z|}
\]

Test the charge symmetry of the interaction
Mirror symmetry is (slightly) broken

Isospin symmetry breakdown manifests in the MED.
An efficient observatory for a direct insight into nuclear structure properties.
Measuring MED

What can we learn from them?

They contain a richness of information about spin-dependent structural phenomena

We measure **nuclear** structure features:

- How the nucleus generates its angular momentum
- Evolution of radii (deformation) along a rotational band
- Learn about the configuration of the states
- Isospin non-conserving terms of the interaction
- Estimate the neutron skin

Can we reproduce theoretically such small energy differences?
MED and nucleon spatial correlations

Shifts between the excitation energies of the mirror pair indicate the type of nucleons that are aligning.
MED and nucleon alignment

D.D. Warner, M.A. Bentley and P. Van Isacker,
MED and Shell model

We start from diagonalizing a nuclear hamiltonian that conserves isospin and treat Coulomb and other eventual isospin symmetry breaking (ISB) contributions perturbatively

A.P. Zuker et al., PRL 89, 142502 (2002)
M.A. Bentley and S.M. Lenzi, PPNP 59, 497 (2007)
The Coulomb effects

\[ V_C = V_{CM} + V_{Cm} \]

\( V_{CM} \) Multipole part of the Coulomb energy

\( V_{Cm} \) monopole part of the Coulomb energy

radial effect: radius changes with J

\( \ell \cdot \ell \) term to account for shell effects

\( \ell \cdot s \) electromagnetic spin-orbit term

Between valence protons only

change the single-particle energies
The radial effect with the shell model

The radius of a nucleus depends on the occupation of the different orbitals and in the fp shell, the \( p \) orbits have larger radius than \( f \) orbits. The radial term will depend on the change of occupation of the \( p \) orbitals as a function of \( J \).

\[
V_{cm}(J) = 2|T_z|a_m \left\langle \frac{\Delta z_p + \Delta n_p}{2} \right\rangle_J
\]

\( \Delta z \) and \( \Delta n \) are the number of protons and neutrons in the \( p \) orbits, relative to the g.s. (\( J=0 \)).

\( a_m \) is not a free parameter but can be estimated from experimental data:

The radial parameter amounts to \( a_m \sim 200 \) keV for nuclei in the \( f_{7/2} \) shell.
Are Coulomb corrections enough?

Another term of “nuclear” nature is needed, but it has to be big!
An empirical ISB interaction

Based on the experimental MED of $A=42$, Zuker et al. (PRL 89, 142502 (2002)) suggested an empirical isovector correction of «nuclear» origin to account for the MED's in nuclei of the $f_{7/2}$ shell.

The ansatz has been further developed and generalized to all shells.

\[
V_B = \left[ \pi f_{7/2}^2 \right]_{J=2} - \left[ \nu f_{7/2}^2 \right]_{J=2} = +100 \text{ keV}
\]

\[
V_B = \left[ \pi f_{7/2}^2 \right]_{J=0} - \left[ \nu f_{7/2}^2 \right]_{J=0} = -100 \text{ keV}
\]

Calculating the MED with SM

\[ MED_{J}^{theo} = \Delta \langle V_{CM} \rangle_{J} + \Delta \langle V_{ls} + V_{ll} \rangle_{J} + \Delta \langle V_{CM} \rangle_{J} + \Delta \langle V_{B} \rangle_{J} \]

- \( V_{CM} \): gives information on the nucleon alignment or recoupling
- \( V_{Cm} \): gives information on changes in the nuclear radius

Important contribution from the ISB \( V_{B} \) term:
- of the same order as the Coulomb contributions

A. P. Zuker et al., PRL 89, 142502 (2002)

\( ^{49}\text{Mn}-^{49}\text{Cr} \)
MED in $T=1/2$ states

Very good quantitative description of data without free parameters

MED in T=1 states

Same parameterization for the whole f_{7/2} shell

M.A. Bentley and S.M. Lenzi,

Silvia Lenzi - Colloque GANIL 2021 - September 27th, 2021
ISB from realistic interactions

Charge-dependent realistic interactions do not reproduce the MED and TED

\[ ^{54}_{28}\text{Ni} - ^{54}_{26}\text{Fe} \]

MED (keV)

0.65*AV18

Exp

The charge-symmetry breaking in the nucleon-nucleon interaction is poorly known

A. Gadea et al., PRL 97, 152501 (2006)

W.E. Ormand et al., PRC 96, 024323 (2017)
MED in $T=3/2$ and $T=2$

Knockout reactions at relativistic energy
Need high efficiency of gamma detection for large T mirror pairs

S. Pigliapoco et al., to be published
A=79: the highest $T=1/2$ MED observed

Measures at NSCL (MSU) using GRETINA
High-resolution measurements


From M. Bentley lectures at Euroschool 2021
The case of $^{73}\text{Sr}$-$^{73}\text{Br}$

A $J^m = 5/2^-$ spin assignment to the g.s. of $^{73}\text{Sr}$ is needed to explain the proton-emission pattern observed from the $T = 3/2$ IAS in $^{73}\text{Rb}$

the ground state of $^{73}\text{Sr}$ differs from that of its mirror $^{73}\text{Br}$

E. M. Ho, A. M. Rogers et al.,
Nature 580 52 (2020)
Interpretation of the data

The data are interpreted by means of the Gamow coupled-channel calculations, considering prolate and oblate deformations.

Conclusion: g.s. $J^\pi = 5/2^-$ in $^{73}\text{Sr}$, but the inversion of the states in $^{73}\text{Br}$ is not reproduced.
N\textasciitilde{}Z nuclei in the A\textasciitilde{}70-80 region

Around the N=Z line quadrupole correlations are dominant

The onset of the different modes of quadrupole collectivity depends on the structure of the spherical mean field

Prolate and oblate shapes coexist

Experimentally it may happen that ISB effects exchange the order of nearby states of different intrinsic structure in mirror nuclei
Quadrupole collectivity and Shell model

Deformed structures can be reproduced by shell model provided a suitable (symmetry-based) model space is considered.

\[
\begin{align*}
\text{Lowest} & \quad \Delta j = \Delta l = 2 \\
\text{Full shell} & \quad \text{except the larger } j\text{-orbit}
\end{align*}
\]

In this mass region the model space that contains the relevant degrees of freedom for the development of quadupole correlations is the \textit{fpgds}


These calculations are not restricted to axially symmetric shapes.
The full MED for $A=73$, $T=3/2$

\[ MED_J = E_J^*({}^{73}\text{Sr}) - E_J^*({}^{73}\text{Br}) \]

\[ = \Delta\langle V_{CM} \rangle_J + \Delta\langle V_{ls+ll} \rangle_J + \langle V_{cm} \rangle_J + \langle V_B \rangle_J \]

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>$V_{CM}$</th>
<th>$V_{ls+ll}$</th>
<th>$V_{cm}$</th>
<th>$V_B$</th>
<th>MED</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2$^-$</td>
<td>11 keV</td>
<td>23 keV</td>
<td>-16 keV</td>
<td>25 keV</td>
<td>43 keV</td>
</tr>
</tbody>
</table>

Since the excitation energy of the $1/2^-$ state in $^{73}\text{Sr}$ is not yet known we have just a lower experimental limit for the MED of this state: $MED \geq 27$ keV.
MED and the ISB in $^{73}\text{Sr}-^{73}\text{Br}$

Experimental data

E. M. Ho, A. M. Rogers et al., Nature 580 52 (2020)

Putting together the calculated MED and the data in $^{73}\text{Br}$, the inversion of the states in $^{73}\text{Sr}$ is obtained.

We predict the $1/2^-$ state in $^{73}\text{Sr}$ at $\sim 16$ keV.

To account for the experimental MED lower limit, an ISB contribution of $V_B \geq 10$ keV is needed.

S.M. Lenzi, A. Poves and A. O. Macchiavelli, PRC 031302(R) (2020)
Isospin symmetry breaking from electromagnetic transitions
Isospin mixing

Electromagnetic transitions between analogue states in isospin multiplets follow some selection rules:

For E1 transitions:
- Analogue E1 transitions in mirror nuclei must have the same strength
  D. Tonev et al., Physics Letters B 821 (2021) 136603
- $\Delta T = 0$ E1 transitions in self-conjugate nuclei are forbidden

For E2 transitions:
- Electric quadrupole matrix elements between $\Delta T = 0$ states are linear in $T_z$ within an isobaric multiplet

These selections rules assume isospin symmetry. The Coulomb field, however, introduces (perturbatively) an asymmetry.

The small isospin impurity admixtures that it causes can allow transitions otherwise forbidden or induce violations to these rules.
Isospin symmetry in $T=1$ triplets

The selection rule for E2 transitions implies that in an isobaric triplet, the proton matrix element between two states of the same isospin has to be linear in $T_z$

$$M_p = \frac{2}{\sqrt{2}} (2J_i + 1) B(\text{E2}, J_i \rightarrow J_f)$$

$$M_p (T_z) = \frac{1}{2} (M_0 - M_1 T_z)$$

This can be tested for the case of the $B(\text{E2}, 2^+ \rightarrow 0^+)$

Test of isospin symmetry in $T=1$ triplets

Deviations from the linear dependence of $M_p$ may arise from isospin mixing

This may occur in the $T_z=0$ odd-odd isobar where $T=0$ and $T=1$ states coexist at low excitation energy.

These measurements are challenging and systematic errors are usually large ($\sim 10\%$)

The $B(E2)$ may be determined by lifetime measurements or by Coulomb excitation and this, in general, requires radioactive beams.

$^{50}\text{Mn}$, $^{50}\text{Cr}$: M.M. Giles et al., Phys Rev C99, 044317 (2019)  

$^{50}\text{Fe}$: K. Yamada et al.,  
The case of the $A=70$ triplet

Relativistic Coulomb excitation of $^{70}$Kr (15 pps) produced by projectile fragmentation at RIBF (RIKEN)

The $>3\sigma$ deviation from the linear behaviour is explained in terms of:
shape of $^{70}$Kr different from the other isobars
(shape coexistence is expected near the ground state)

K. Wimmer et al, Phys Rev Lett 126, 072501 (2021)
Shell model analysis for $A=70$

We calculate the excitation energy and the $B(E2)$ values using two different valence spaces and incorporate the Coulomb interaction.

- **JUN45**: M. Honma, T. Otsuka, T. Mizusaki, M. Hjorth-Jensen, PRC 80, 064323 (2009)

**JUN45** gives 1p$_{3/2}$, 1p$_{1/2}$, 0f$_{5/2}$, 0g$_{9/2}$

**JUN45+LNPS** gives 1p$_{3/2}$, 1p$_{1/2}$, 0f$_{5/2}$, 0g$_{9/2}$, 1d$_{5/2}$

JUN45+LNPS gives better spectroscopy than JUN45

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The results for A=70

The triplet does not show a well defined shape

A Kumar* analysis gives for the intrinsic deformation parameters:

\[ \beta = 0.22 \pm 0.05 \]
\[ \gamma = 32^\circ (+28^\circ - 24^\circ) \]

Our results do not support a change of shape between \(^{70}\text{Kr}\) and \(^{70}\text{Se}\)

We calculate the \(M_p\) matrix elements using two sets of effective charges:

Standard values (ST): \(e_p = 1.5e, e_n = 0.5e\)

Dufour-Zuker (DZ): \(e_p = 1.31e, e_n = 0.46e\)


The $^{70}$Br scenario

The deviation of the B(E2) in $^{70}$Br from the straight line determined by the even-even isobars could be due to:

- isospin mixing
- an unobserved $1^+ T=0$ state below the $2^+ T=1$

Exclusion plot to determine the excitation energy of an hypothetical $1^+$ state that could explain this anomaly.

Conclusions

MED put in evidence the isospin symmetry-breaking of the nuclear interaction that is not well reproduced by realistic interactions. They are sensitive to the nuclear structure and therefore constitute a very powerful tool to understand several nuclear properties.

Several experiments are being performed at different facilities using stable and radioactive beams to study mirror nuclei far from stability and to characterize the ISB interaction.

To come to solid conclusions it important to have a reliable theoretical description, including the relevant degrees of freedom of the system.
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